Technical Report

A Power Model for DC-DC Boost Converters Operating in PFM Mode

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1 Introduction

Next generation of computing is going to be outside of the traditional stationary computing realm [1]. In the future paradigm, many non-stationary objects around us sense and actuate on the environment while they are connected to each other via internet. During the last few years, the number of these devices has been growing rapidly [2]. This is making an explosion of small computing platforms for commercial, consumer, and industrial use cases [3].

This revolution is mainly an outcome of the advancements in the field of Internet of Things (IoT). This term was first mentioned in 1999 by Kevin Ashton within the supply chain management context. The early versions of this concept were using Radio Frequency Identification (RFID) in a form of sensor networks [1]. Nonetheless, this concept had expanded its territory into multiple other applications as well. Nowadays, IoT devices have a wide range, from fitness tracker and health monitors to drones, smart grids and logistics [3].

The overall concept of IoT is based on the communication (mainly through the internet) between multiple entities which are generalised as *things*. According to the diversity of the application fields, large number of entities are considered as *things*. From simple one-bit sensors to complex robots. Even some concepts consider human being as an entity within an IoT system. This leads into ambiguity of the definition for objects. Consequently, no unified definition for *things* is accepted among different communities. However, Cyber Physical Systems (CPS) as embedded devices with communication capabilities would fit into most (if not all) of them.

Independent from the application field, cyber physical and embedded devices collect information from the environment (sensing), analyse these data (intelligence) and in some cases interact with the physical world (actuation). Communication possibilities enable them to share these information with others and interact with the environment [1].

2 Motivation

In spite of differences in the size, specification and application domains of the IoT devices, they have all one common aspect. They all require one form of power supply to enable internal electric and electronic circuitry. Communication as a key aspect of IoT devices, is an energy intensive task as well. Not only electric power is needed for the communication, but also the normal operation of these objects requires energy even in their idle mode.

Although, intensive actions such as computation or communication were traditionally very limited on non-stationary devices, developments in the field of cyber-physical and embedded systems, makes the realization of smart objects running on low power possible [4].

Current battery technologies which have an acceptable power density, is considered as a common source of power supply for the IoT entities. However, still there are challenges for the use of batteries as the sole power supply which three of them are explained here.

At first, battery life is the bottleneck which limits the entities' life span. Although it is possible to use larger battery size, limitations on the size and weight of most IoT entities make their implementation hard, if not impossible.

Second, changing the batteries increases the maintenance cost of an IoT systems [5] and can even disgrace the whole feasibility of the solution in some cases.

Moreover, IoT entities operate in dynamic environments with continuous changing condition [2]. For instance, CPS objects used in the field of materials handling and logistic fulfil the modularity and flexibility requirements for the realization of smart objects [6]. However, this application field may cause a device be in an extreme remote location for a long period. This makes access to the device much harder and even impossible in some cases. Hence, maintenance and running cost of the system will be higher. Furthermore, this will reduce the reliability of the system since lack of battery change or late access will lead into data loss.

2.1 Energy Harvesting

A method to tackle the battery issues of the IoT devices is the use of an energy harvesting system in combination with a rechargeable battery. Although different type of harvesting techniques are available, the general concept of the energy harvesting is considered as a promising option to degrade battery replacement challenges [5].

An example of such a solution is the intelligent Bin (inBin), shown in Figure 1. It is a materials handling unit which uses photovoltaic energy harvesting to have minimum mobility restrictions and fulfil all its functions as an energy-neutral IoT entity [7].

Figure 1: inBin: an energy-neutral IoT device for the materials handling application

2.2 Voltage Matching

Although operational condition of the energy storage is mostly predefined, provided power from the energy harvesting modules changes according to the operational and environmental condition. In spite of very few solutions with a matched voltage[1] between energy harvester and storage, most combinations have different operational voltage between the harvester and the storage. Consequently, some form of converter has to be mounted in between them (such as shown in Figure 2) to match the voltage levels.

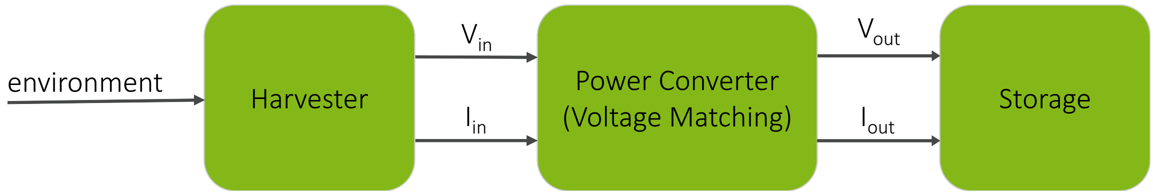


Figure 2: A schematic presentation of a harvesting system with a voltage converter

Moreover, in some cases such as vibrating or wind harvesters, the generated voltage has an AC form and a rectifier is needed as well to convert the delivered power into DC with the proper level before delivering it to the energy storage.

In addition, most of energy harvesters have a internal dynamic behaviour. Therefore, they have an optimal working point for the voltage, current and power. An intelligent converter not only would help to match the voltage levels, but also can set the harvester parameters in a way that the maximum power is harvested.

Within the IoT application, harvesters are mostly small scale and their generate power is in low levels. Therefore, the operational voltage would be mostly smaller than the optimal voltage level of the storage system. Consequently, some kind of up-scaling voltage converter (also known as boost converter) is necessary.

2.3 DC-DC Conversion Techniques

Multiple methods are available for the DC-DC conversion. However, techniques which are based on switching are more desired because of their good efficiency. A switched based DC-DC converters is mostly made of a switch that toggles between charging and discharging an inductor as the active component in the converter. According to the switching scheme, different types of converters are available. Moreover, placement and configuration of the switch(es) and active component defines the direction of the voltage scaling. It is possible to build scale-up (boost), down (buck), and up-down (buck-boost) converters.

Pulse Width Modulation (PWM) is the most common switched based converter technique which is commonly known and is widely spread both in academia and industry. This technique controls the output voltage by manipulating the width of the output signal. PWM converters can be made to have a high efficiency when they work at the full load. However, while many of its losses are not dependent to the output current (load), at light loads it dissipates a significant portion of the power. When the operational point (load)

[1] This has to be accurately considered during the design and component selection phase. In addition, optimal operational working points would be limited.

reduces, the relation between the PWM power dissipation to the load grows and becomes more significant, thereupon reducing the efficiency [8].

Pulse Frequency Modulation (PFM) is a converter control scheme which reaches high efficiency for a wide load range [8]. This method uses short bursts of power to the output and manipulates the overall output power by controlling the frequency of these bursts. Therefore, for low loads it only reduces the frequency of switching and power dissipation remains small compared to the light load.

Within the IoT application with the energy-neutral perspective, most of the harvesters are able to deliver a very limited amount of power. Therefore, using a PFM controller for the conversion of power from harvester to the energy storage is of big benefit.

3 PFM Work Principle

A PFM controlled DC-DC converter manipulates the frequency of very short power bursts to the output. However, an output signal which is not flat and has different AC component is not desired. To provide a more smooth output, a filter capacitor is necessary to feed the output during the time that no power is directly switched out. This capacitor gets charged again during the active period when the energy bursts are available. A schematic representation of this concept is depicted in Figure 3.

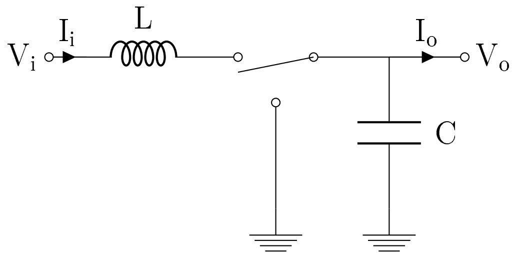


Figure 3: An ideal one switch boost converter

Switched signals and voltages of such a circuit when the switching is done based on PFM scheme would be as in Figure 4.

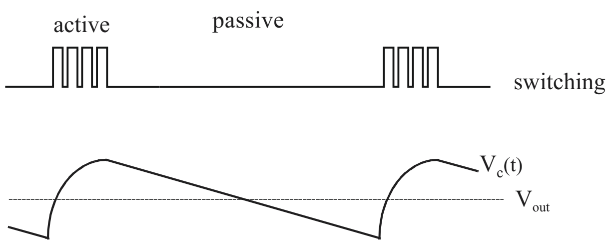


Figure 4: Switching signal of a DC-DC converter in PFM mode and its voltages

A buck converter can be easily built by mounting the inductor after the switch at the output side. Buck converter is very common for supplying different loads from a battery and is the most wanted type of dc-dc converter in the classical design; and consequently is well analysed. However, as mentioned before, within the IoT applications boost converter is necessary and is analysed hereafter.

4 DC-DC Boost PFM converter

In reality, a DC-DC converter is built using two switches. Consequently, a boost converter with the inductor at its input side before the switches would be as shown in Figure 5.

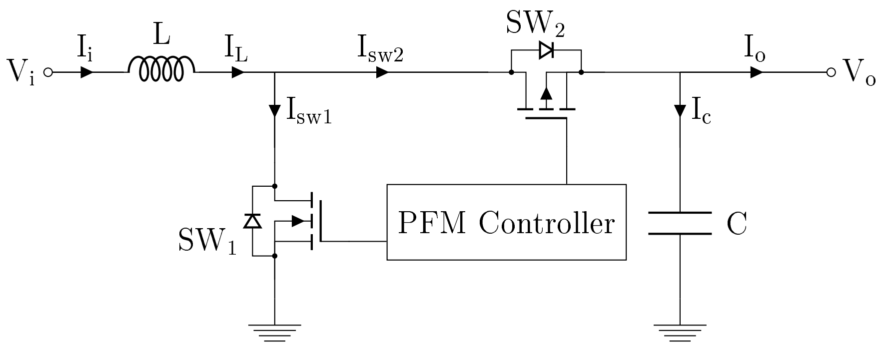


Figure 5: Schematic overview of an ideal boost converter with PFM controller

Although ideal switching makes only two switch combinations as shown in Figure 3, a semiconductor based converter has one more operational condition. According to the state of the switches, three phases are possible which are mentioned in Table 1. A state which both switches are close is not feasible while it causes to short circuit the capacitor filter and also, the output would be short circuit as well with the voltage value zero.

Table 1: Different switching states for a semiconductor based switching converter

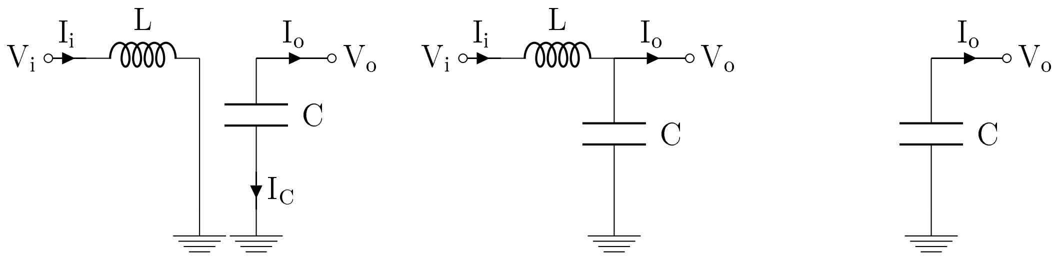
state	φ_1	φ_2	φ_3
SW_1	close	open	open
SW_2	open	close	open

According to the status of the switches in each state, the overall circuit can be simplified into one or two sub-circuits as shown in Figure 5.

Since the inductor is playing the charging characteristic of the converter to boost power to the output side, analysing its current is necessary. During the charging phase (φ_1) this current can be explained using a first order differential equation as:

$$L\frac{dI_L(t)}{dt} = V_i(t) \tag{1}$$

$$I_o(t) = C\frac{dV_o(t)}{dt} = -I_c(t) \tag{2}$$



(a) in φ_1 (b) in φ_2 (c) in φ_3

Figure 6: Boost converter circuit in each operational phase

During the second phase (φ_2), these relations would be as:

$$L\frac{dI_L(t)}{dt} = V_i(t) - V_o(t) \tag{3}$$

$$I_o = I_L - I_C = I_L - C\frac{dV_o(t)}{dt} \tag{4}$$

During the third phase (φ_3), the output current is only fed by the filter capacitor. Therefore:

$$I_L = 0 \tag{5}$$

$$I_o = C\frac{dV_o(t)}{dt} \tag{6}$$

For a better understanding of the converter operation, a simplified version of signals during the two periods are presented in Figure 7.

As explained in the differential expressions, voltage and current of the capacitor and inductor have an exponentially form. However, these signals are linearised in Figure 7 to simplify the analysis. Using these simplified signals and (1), duration of the φ_1 would be:

$$\frac{V_i}{L} = \frac{\Delta I_L}{\Delta t} = \frac{I_p}{t_{\varphi_1}} \Longrightarrow t_{\varphi_1} = \frac{I_p \cdot L}{V_i} \tag{7}$$

where I_p is the maximum allowed current by controller to go through the inductor.

Using (3), the duration of the second phase (φ_2) can be defined as:

$$\frac{V_i - V_o}{L} = \frac{\Delta I_L}{\Delta t} = \frac{-I_p}{t_{\varphi_2}} \Longrightarrow t_{\varphi_2} = \frac{I_p \cdot L}{V_o - V_i} \tag{8}$$

Unfortunately, the differential representation of the circuit in the third phase does not provide any further information about the timing period of this state. However, the active time of the inductor can be represented using (7) and (8) as:

$$t_{\varphi_1} + t_{\varphi_2} = I_p \cdot L\left(\frac{V_o}{V_i\,(V_o - V_i)}\right) \tag{9}$$

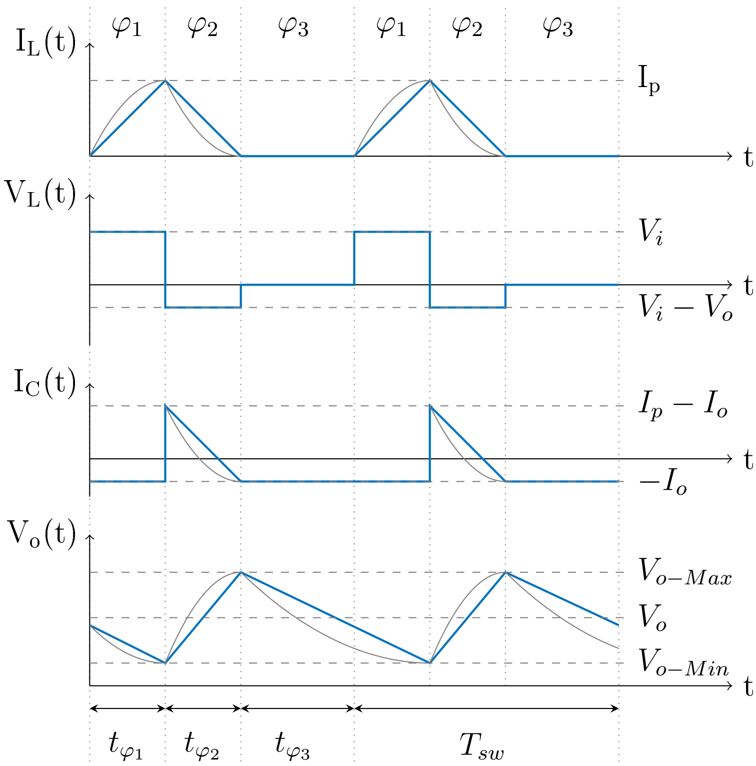


Figure 7: PFM signals

As can be seen from signals in Figure 7, this boost converter is a system which goes back into its starting point at the end of each cycle and repeats its signals [2]. Therefore, instead of a dynamic representation, the Periodic Steady State (PSS) can be analysed for the average steady state behaviour.

Within the PSS, the average input current to the converter is the average inductor current. This current can be simply calculated using the signals in Figure 7. Hence, the average inductor current ($\widetilde{I_L}$) would be:

$$\widetilde{I_L} = \frac{1}{T_{sw}} \left(t_{\varphi_1} \cdot I_{L1} + t_{\varphi_2} \cdot I_{L2} + t_{\varphi_3} \cdot I_{L3} \right) \tag{10}$$

Since inductor current during the φ_3 is zero, using (7)-(9), the PSS inductor current would be defined as:

$$\widetilde{I_L} = \frac{I_p^2 \cdot L}{2 \cdot T_{sw}} \times \frac{V_o}{V_i \left(V_o - V_i \right)} \tag{11}$$

In the PSS, this current is also the input current to the converter. Therefore, we have:

$$I_i = \widetilde{I_L} = \frac{I_p^2 \cdot L}{2 \cdot T_{sw}} \times \frac{V_o}{V_i \left(V_o - V_i \right)} \tag{12}$$

[2] only if the input/output condition do not change.

On the other hand, the PSS inductor current can be graphically calculated by averaging it from Figure 7, leading to:

$$\widetilde{I}_L = \frac{1}{T_{sw}} \left(\frac{I_p}{2} \left(t_{\varphi_1} + t_{\varphi_2} \right) \right) \tag{13}$$

According to the (13), the switching frequency would be:

$$f_{sw} = \frac{1}{T_{sw}} = \frac{2 \cdot \widetilde{I}_L}{I_p \left(t_{\varphi_1} + t_{\varphi_2} \right)} \tag{14}$$

Using (13) or (14), the cycle duration can be explained by:

$$T_{sw} = t_{\varphi_1} + t_{\varphi_2} + t_{\varphi_3} = \frac{I_p}{2\widetilde{I}_L} \left(t_{\varphi_1} + t_{\varphi_2} \right) \tag{15}$$

Using (15) and the fact that a complete cycle is the sum of inductor's active time ($t_{\varphi_1}+t_{\varphi_2}$) and passive time ($t_{\varphi_3}$), the time that the converter remains in the φ_3 would be:

$$t_{\varphi_3} = \frac{I_p - 2\widetilde{I}_L}{2\widetilde{I}_L} \left(t_{\varphi_1} + t_{\varphi_2} \right) \tag{16}$$

The inductor volt-second balance principle declares that in the steady state, the energy stored and delivered by it are equal. Therefore, the net current change of the inductor during one period is zero[3]. Consequently:

$$I_L(T_{sw}) - I_L(0) = 0 = \frac{1}{L} \int_0^{T_{sw}} V_L(t) \mathrm{d}t \longrightarrow V_i \cdot t_{\varphi_1} + (V_i - V_o) \cdot t_{\varphi_2} = 0 \tag{17}$$

which leads to:

$$\frac{V_o}{V_i} = \frac{t_{\varphi_1} + t_{\varphi_2}}{t_{\varphi_2}} \tag{18}$$

hence, in the ideal form of a converter without any loss, the input/output current relation is:

$$I_o = \frac{t_{\varphi_2}}{t_{\varphi_1} + t_{\varphi_2}} I_i = \frac{t_{\varphi_2}}{t_{\varphi_1} + t_{\varphi_2}} \widetilde{I}_L \tag{19}$$

5 Non-ideal Boost PFM Converter

Although these formulation would represent the converter's signals in the ideal form, some part of the input energy is lost within the converter. Therefore, the power balance can be written as:

$$P_o = P_i - P_l \tag{20}$$

[3]with this consideration that its value does not change

when P_o is the output power, P_i the input power and P_l presents the lost power. The lost portion of power is made of multiple dissipations. However, it can be simplified into three terms as:

$$P_l = P_{controller} + P_{switching} + P_{conduction} \tag{21}$$

$P_{controller}$ is the power which the internal circuitry of the PFM controller needs to operate and can be simply presented as:

$$P_{controller} = V_i \cdot I_{controller} \tag{22}$$

$P_{switching}$ is that portion of power which is lost because of switching and can be divided into two sub-parts. One section is inductor core loss and the second part is because of the switching gates of transistors.

According to the [9], inductor core loss is related to the switching frequency, peak current square, in addition to a proportionality factor. This can be formulated as:

$$P_{core} = k_{core} \cdot I_p^2 \cdot f_{sw} \tag{23}$$

Second part of the switching dissipation, is the loss from the switches' gates. Consequently, more switching will increase this loss and it is a function of the switching frequency. Moreover, this dissipation is roughly proportional to the input voltage and the gate charge of the switch. Hence, it can be presented as:

$$P_{gate} = (Q_{sw1} + Q_{sw2}) \cdot V_i \cdot f_{sw} \tag{24}$$

when the gate charge (Q_{sw}) is a constant value defined based on the gate width in on and off mode [10]. Therefore, for a DC-DC converter this can be simplified by considering a sum of the gates' charges as $Q_{sw} = Q_{sw1} + Q_{sw2}$. Then, the gate power loss will be:

$$P_{gate} = Q_{sw} \cdot V_i \cdot f_{sw} \tag{25}$$

$P_{conduction}$ is the lost power because of the internal resistance of components during the time that the current passes through them. Therefore, an ideal converter circuit (as shown in Figure 5) has to be modified to include these resistances as well and will be as shown in Figure 8.

Since each resistance is not always in the circuit, each phase of the circuit has to be analysed separately. This can be seen in Fig. 9.

Each resistor is active only in some of the phases, these portion of time has to be calculated according to the overall cycle time. Hence, time ratios has to be calculated which are as:

$$\tau_{\varphi_1} = \frac{t_{\varphi_1}}{T_{sw}} = \frac{2\widetilde{I_L}}{I_p} \times \frac{V_o - V_i}{V_o} \tag{26}$$

$$\tau_{\varphi_2} = \frac{t_{\varphi_1}}{T_{sw}} = \frac{2\widetilde{I_L}}{I_p} \times \frac{V_i}{V_o} \tag{27}$$

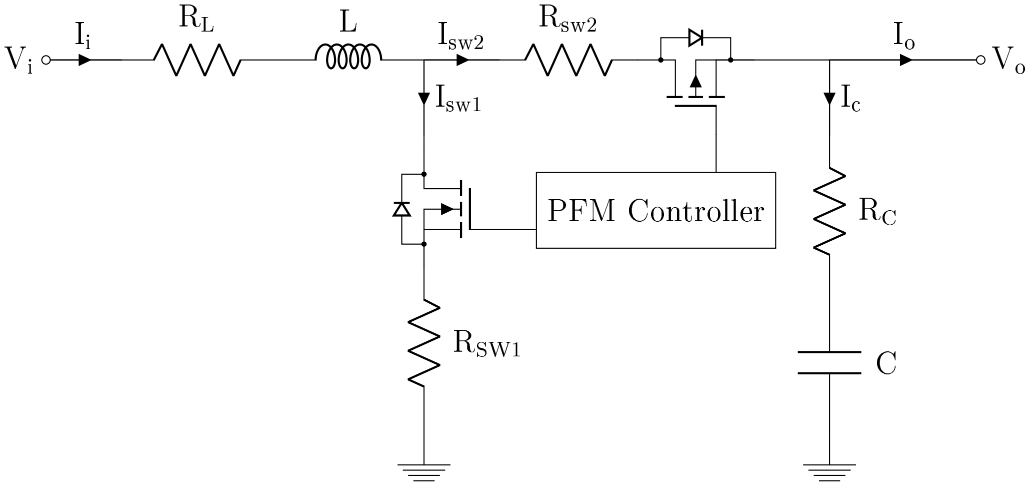
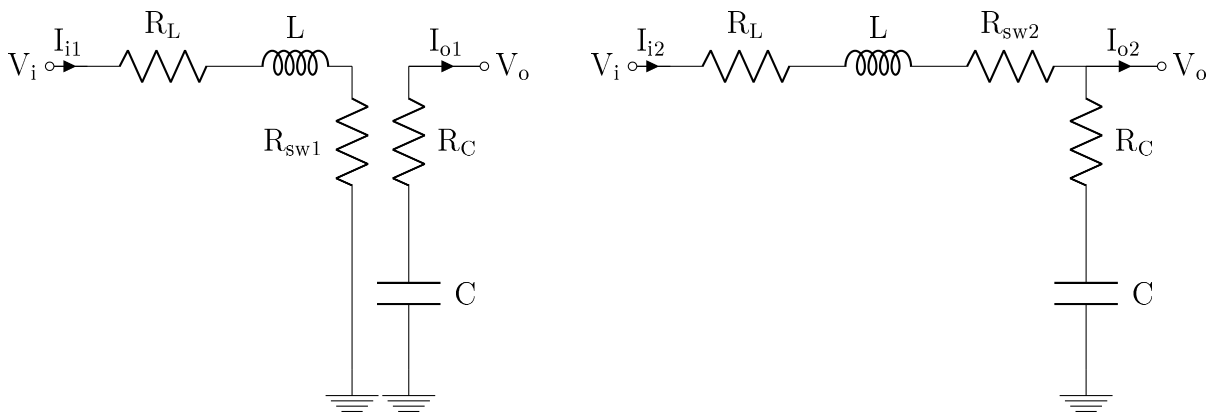


Figure 8: Overview of the PFM boost converter, including non-ideal parameters



(a) in φ_1

(b) in φ_2

Figure 9: Non-ideal PFM boost converter circuit in each operational phase

$$\tau_{\varphi_{12}} = \frac{t_{\varphi_1} + t_{\varphi_2}}{T_{sw}} = \frac{2\widetilde{I_L}}{I_p} \tag{28}$$

Consequently, the power loss during the conduction time by resistances can be calculated using:

$$\begin{aligned} P_{conduction} = \tau_{\varphi_{12}} \Bigg[& \left(\frac{I_p}{2}\right)^2 (\tau_{\varphi_1} R_{sw1} + \tau_{\varphi_2} R_{sw2} + \tau_{\varphi_{12}} R_L) \\ & + \frac{1}{3}\left(\frac{I_p}{2}\right)^2 (\tau_{\varphi_1} R_{sw1} + \tau_{\varphi_2} R_{sw2} + \tau_{\varphi_{12}} R_L + R_C) \Bigg] \end{aligned} \tag{29}$$

As can be seen in (29), the conduction dissipation is made of a DC component (first line) and an AC part which are calculated separately.

6 Conclusion

After a short explanation of the demand for the IoT systems and their need for long lasting operation in an energy-neutral manner, PFM DC-DC converter is presented as a solution to convert the energy provided by an energy harvesting module into the energy storage. This combination will help IoT devices to last longer without the need for battery change and maintenance.

A general model for the DC-DC boost converter with the PFM control technique is provided. After analytical modelling of this system in the ideal form, a non-ideal model is provided. In addition, a model for the power losses including dissipations by controller, switching, and conduction is explained.

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